Turbulence Limits in Rotating Plasmas for Isotope Separation

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With the aim of using rotating plasmas (driven by $\boldsymbol{j} \times \boldsymbol{B}$ or $\boldsymbol{E} \times \boldsymbol{B}$ forces) for isotope separation, several experimental configurations have been studied and theory has been developed with respect to numerical models, the Alfvén ionization limit, D'Angelo's shear instability, while turbulence was suggested qualitatively only. A more detailed study of the limit of laminar flow is presented taking into account the magnetic anisotropy of viscosity and interaction with neutrals. In all cases of experiments, the Reynold's numbers are below critical values of 10^4 by at least a factor 100. The pessimistic conclusions of Boeschoten for his experiment can therefore not be shared with respect to the turbulence problems. The published cases of isotope separation by rotating plasmas are then discussed extensively.

1. Introduction

It was probably Slepian [1], who in 1942 first proposed an isotope separator, utilising the high rotational velocities observed in early plasma experiments. This Ionic Centrifuge was based on the difference in centrifugal forces which are experienced by ions of different mass, rotating in crossed electric and magnetic fields. This was used to separate uranium isotopes. However, the idea was not enlarged upon because of the growing interest in the diffusion process which is still being used. We restrict our discussions on plasmas of medium density [2].

In a plasma centrifuge the ionized particles are set in rotation by means of the $\boldsymbol{j} \times \boldsymbol{B}$ Lorentz force, which may be generated by two different combinations of current and magnetic field components, $j_r B_z$ or $j_z B_r$ [3]. In the 1960s Millar and Watson-Munro [4], Boeschoten [4a], Hora and Karger [5] and Bonnevier [6] revived the idea, including $\boldsymbol{J} \times \boldsymbol{B}$ and $\boldsymbol{E} \times \boldsymbol{B}$ drifts. Since that time, the possibility of using plasma centrifuges has been studied intensively, e.g. by James and Simpson [7], Nathrath [8], Wijnakker et al. [9] and Kaneko [10]. The main advantage of a plasma centrifuge above a mechanical one is the much higher separative powers which are in principle attainable (Lehnert) [11].

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The experiments performed to date could be put into two classes. In the first class, the scheme of a nearly fully ionised plasma at a low background gas pressure is used, e.g. Bonnevier [12], Boeschoten [3], [13]. Here, very high rotational velocities can be achieved, although this is at the expense of a large electrical power input needed to sustain the plasma. Perhaps more successful is the approach whereby a partially ionised plasma is used to drive a neutral gas into rotation via collisional processes. The economics of this scheme appear to be brighter due to the smaller power input because of the smaller part of the gas that has to be ionized. The rotational velocity is not as high as in the fully ionized case, however. Experiments utilising this scheme have dominantly been done using noble gases, e.g. helium, argon, krypton [14] and neon [15], [16]. Also hydrogen [17] has been studied along with uranium [18] with the hope that this will eventually lead to a more economic isotope enrichment method than those currently employed.

Models based on separation within the neutral particles as well as models based on separation within the ions have been proposed and considered theoretically. The majority of these apply the MHD equations and are solved (usually through numerical methods) to match the particular boundary conditions.

2. Theoretical Analysis

Various analytical approaches may be employed. For example, Bonnevier [6] and Lehnert [19] developed theories based on the conservation of momen-

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tum for each of the species in the case that the gas is fully ionised. Another approach is to solve the Navier-Stokes equation for D.C. discharges as was done by Klüber [20] and Wilhelm and Hong [21]. James and Simpson [22] gave a time dependent treatment of the Navier-Stokes equation.

The case of partial ionisation is more difficult to examine. Both Lehnert [11] and Vrba [23] have set up a model based on the conservation equations for mass, charge, momentum, energy and the generalised Ohm's law. Wijnakker et al. [9], for the analysis of their experiments which utilisedan ionisation of only a few percent, have chosen to adapt the single fluid MHD theory of Wilhelm and Hong, as this seemed to describe their system adequately.

The complex interdependence of the various parameters that describe a rotating plasma make it necessary to introduce a series of simplifications. Calculations are usually carried out for a simplified cylinder geometry, and for the most part only the radial dependence of the various quantities such as velocity, pressure, temperature and electric field are obtained.

The flow in the system is assumed to be purely azimuthal i.e. secondary flows in radial and/or axial direction are neglected. This means that the plasma behaves as an incompressible fluid ($\nabla \cdot \boldsymbol{v} = 0$) and that density gradients $\nabla \varrho$ (pressure gradients $\nabla \varrho$) are only present in the radial direction.

Perhaps the simplest configuration to treat is that of a fully ionised plasma rotating in a cylindrical annulus. Okada and Dodo [24] solved this for the velocity and density distributions under the assumption of no axial flows and uniform temperature T and electrical conductivity σ. Induced magnetic fields and mass currents are neglected, and the forces acting on the medium are the viscous friction and the electromagnetic force (the electric field is applied in the radial direction and the magnetic field is axial).

The conservation equations for fluid mass and momentum at uniform temperature are

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot (\varrho \boldsymbol{v}) = 0\,, \tag{1}$$

$$\varrho\left(\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\,\boldsymbol{v}\right) = -\nabla p + \boldsymbol{j} \times \boldsymbol{B} + \mu\left(\nabla^2 \boldsymbol{v} + \frac{1}{3}\nabla(\nabla \cdot \boldsymbol{v})\right) \tag{2}$$

where μ is the viscosity.

Ohm's law may be written as

$$j = \sigma\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} - \frac{1}{e\,n_e}\,\boldsymbol{j} \times \boldsymbol{B}\right). \tag{3}$$

These three equations, together with the equation of state

$$p = \varrho\, k\, T/m \tag{4}$$

may then be solved to obtain the $\boldsymbol{v}$, p, ϱ and $\boldsymbol{E}$ distributions.

Okada and Dodo solved these to give

$$v_\theta = \frac{B_0 J}{4 \Pi L \mu (r_1^2 - R^2)\, r} \cdot \left(r_1^2 (r^2 - R^2) \ln \frac{r}{r_1} - (r_1^2 - r^2) R^2 \ln \frac{R}{r}\right) \tag{5}$$

for the velocity distribution. B_0 and L are characteristic values for magnetic field and vessel dimension, r_l is at the inner and R at the outer electrode surface.

A general form of the velocity distribution thus derived is shown in Figure 1. The maximum velocity is reached roughly midway between the electrodes. The electromagnetic drift velocity, which is given by the local electric field and the axial field as E_r/B_0, is represented in the figure by a broken line. The drift velocity, which gives the upper limit of the velocity of charged particles, is higher than the rotational velocity (in the case shown) by about 5×10^5 cm/sec.

A more detailed analysis of this simple system was carried out by Thiagarajan and Rohatgi [25]

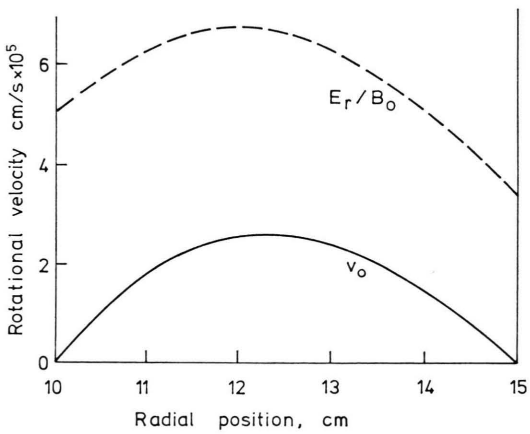

Fig. 1. Radial distribution of rotational velocity for an ionised uranium gas ($r_1 = 10$ cm, $R = 15$ cm, $B_0 = 200$ G, $J = 1.5$ kA, $T = 0.22$ eV) (after Okada et al. [24]).

who examined a collisional plasma in the diffuse discharge regime on the basis of 2-species MHD momentum equations, including Lorentz, pressure, centrifugal, Coriolis and collisional forces. Plasma neutrality and constant and uniform electron and gas temperatures were again assumed; boundary layer effects were neglected.

A numerical analysis, using Argon as the working gas at a pressure of 33 torr was carried out. Species velocity and density distributions exhibited several interesting features, including:

— Coriolis and electron pressure garadient terms need to be included for the simultaneous balance of radial and azimuthal momenta.

— Retrograde motion (in part or whole of the annulus) occurs over a range of $\boldsymbol{B}$-fields for current flowing radially inward.

The balance of a multi-component plasma rotating in an annulus was investigated by Hellsten [26]. For multi-component plasmas, the separation degree of the components is found to depend mainly on ion-ion collisions, whereas the fluxes of the separated ions depend on ion-electron collisions.

The pin cathode-ring anode system was studied by Nathrath [8], and recently an intensive study was done by Wilhelm and Hong [21].

Wilhelm and Hong introduce a number of non-dimensional parameters that characterise the plasma:

The Hartmann number H,

$$H = (\sigma/\mu)^{1/2} B_0 R_0 , \tag{6}$$

which is a measure of the Lorentz force relative to the viscous force.

The magnetic Reynold's number R_m

$$R_\mathrm{m} = \mu R_0 \sigma v , \tag{7}$$

which is a measure of the intensity ratio of the induced and external magnetic fields and can indicate the degree of bending or "dragging along" of the magnetic field lines due to the motion of the fluid through the field. Also, the electron and ion Hall parameters

$$\beta_\mathrm{e} = \omega_\mathrm{e}/\nu_\mathrm{e} ; \quad \beta_\mathrm{i} = \omega_\mathrm{i}/\nu_\mathrm{i} \tag{8}$$

with $\omega_{\mathrm{e,i}}$ the cyclotron frequency and $\nu_{\mathrm{e,i}}$ the collision frequency, may be defined.

This system was solved using (in the magnetogasdynamic approximation) the equation of conserva-

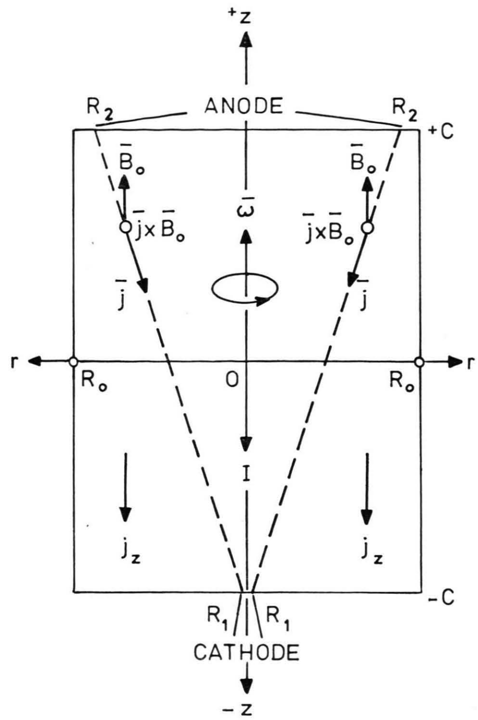

Fig. 2. Scheme of diverging discharge (after Wilhelm and Hong [21]).

tion of electric charge density ($\boldsymbol{\nabla} \cdot \boldsymbol{j} = 0$) and the Navier-Stokes equation, both combined with the generalised Ohm's law and the equation of state. The boundary-value problem was then solved assuming that the plasma does not slip at the side walls $r = R_0$ and the ends $z = \pm c$ (see Fig. 2) and that no current flows into the cylinder wall at $r = R_0$, i.e.

$$\left(\frac{\partial \varphi(r, z)}{\partial r} \right)_{r = R_0} = 0 , \quad -c \leqq z \leqq c . \tag{9}$$

This boundary-value problem is essentially difficult and presents many analytical difficulties (see Wilhelm and Hong).

Finally the configuration of George and Kane [27] is noted (Figure 3). A thorough investigation was done for this immersed magnet scheme.

3. Velocity Limitations and Turbulence

To achieve the desired separation effect for heavy gases with small isotopic differences by centrifugal forces, high velocities are required. In the ultracentrifuge method, the gas is kept rotating due to

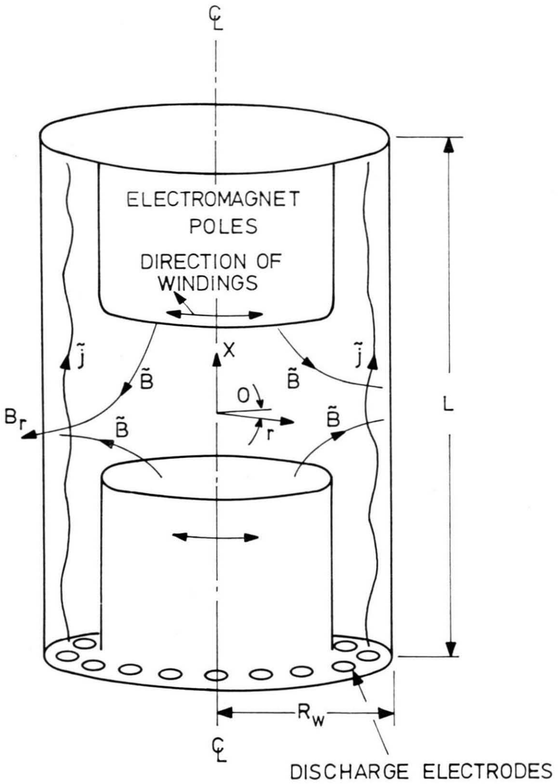

Fig. 3. Immersed magnet chamber geometry (after George and Kane [27]).

the coupling by viscous effects of the wall velocity to the main gas body.

Similarly, the neutral gas in the partially ionized boundary regions of a rotating plasma is usually coupled in an efficient way with surrounding vessel walls. Compared to the high velocities of the rotating plasma, it is therefore nearly at rest with respect to the walls. Alfvén [28] postulated that the differential velocity between the plasma and the neutral gas provides an energy source for ionisation in the partially ionised regions.

This effect was proposed by Alfvén in connection with his theory on the origin of the solar system. Essentially it predicts a rapid ionisation of the neutral gas as soon as the relative motion between plasma and gas exceeds a critical velocity. This is given by

$$v_c = \left(\frac{2\,e\,\varphi_i}{m_i}\right)^{1/2}, \tag{10}$$

where φ_i is the ionisation potential.

For hydrogen this velocity is $\sim 5.12 \times 10^6$ cm s^{-1}, for Argon $\mu \sim 8.72 \times 10^5$ cm s^{-1} and for Uranium $\sim 2.19 \times 10^5$ cm s^{-1}. These roughly correspond to the maximum achieved values in most experiments.

This mechanism is understandable only for the transient state, where neutrals are at rest and the electrons and ions receive a plasma velocity v_c by the external field. If the neutrals are speeded up by collisions, this ionisation will not take place and higher rotation velocities are expected. There are several experiments [29], [30], [31] which support this hypothesis, however the underlying physical mechanism is not fully clear. Among several models [32] which try to explain the critical velocity phenomenon, the most successful ones in the light of recent experiments have turned out to be those which involve collective effects such as turbulent electron heating by means of two-stream instabilities.

Himmel et al. [29], using a homopolar device with magnetic field strengths up to 6×10^3 G and a radial electric field produced by concentric electrodes, showed that within a wide range of parameters the velocity $\boldsymbol{v}$ for various gases agrees with Alfvén's critical velocity $\boldsymbol{v}_c$. This was elaborated then for helium. By varying the neutral gas pressure, a lower and an upper value was found for the critical velocity. Within a range below 5×10^{-4} torr, no self-sustaining discharge could be set up, even with preionisation. A range 5×10^{-4} torr up to 5×10^{-1} torr is marked by the critical velocity. Above 5×10^{-1} torr, the burning voltage of the discharge steadily falls below the critical value.

Another velocity limitation factor could be the departure from laminar flow by excessive turbulence due to the possible high Reynold's numbers achievable. This will be discussed in the next section.

Kelvin-Helmholtz (D'Angelo [33]) or velocity shear instabilities might be expected under certain circumstances. Lehnert [34] reviewed other instability phenomena which affect the separation efficiency. The ionisation instability [35] and other modes associated with the plasma-neutral gas interaction should be taken into account, however there exist stable ranges for most of these modes.

Other contributing factors towards turbulence are the secondary flows and instabilities occurring in the MHD flow between concentric cyclinders as described by Chandrasekhar [36].

George and Kane [37] proposed that a turbulence damping mechanism may exist. This was based on experiments performed by Lykoudis and Gardner [38], who showed that a turbulence damping laminarisation effect arises when magnetic field lines are directed in a ormal direction to the mean velocity flow. However, this was done for incompressible channel flow, and the link to a rotating plasma system may not be strong.

4. Turbulent Flow at High Reynold's Number

In the preceding section, we previewed where several authors suggested qualitatively a turbulence process in the case of rotating plasmas, apart from other instabilities. We are now studying turbulence within the limitations of anistropic viscosity and of ion-ion collisions.

Another possible velocity limiting factor could be the departure from laminar flow caused by excessive turbulence due to the high Reynold's numbers that may be reached under certain conditions. In fact, Boeschoten [13] suggests that a kink in the radial density profile at a radius r_k is the beginning of a turbulent region which provides a transition to the wall.

The Reynold's number, R_e, is proportional to the inverse of the viscosity, i.e.

$$R_e = v\, d\, n_i\, m_i/\mu\,, \tag{11}$$

where n_i is the particle density, d some characteristic length such as the distance from a boundary wall and μ the viscosity.

To begin, the viscosity of a fully ionised gas in the absence of a magnetic field is examined. This is given by (Lehnert [19]):

$$\mu = n_i\, k\, T_i/3\, \nu_{ii}\,, \tag{12}$$

where

$$\nu_{ii} \sim k_1 n_i Z^4 \ln \Lambda / T_i^{3/2} A_i^{1/2} \tag{13}$$

is the ion-ion collision frequency and A_i is the molecular weight of the ions.

k_1 is approximately equal to 2×10^{-2}, but its exact value varies from author to author, e.g. Spitzer [39], Hora [40].

Using Spitzer's value,

$$\mu = 2.21 \times 10^{-15} \cdot (T^{5/2} A_i^{1/2}/Z^4 \ln \Lambda)\ \mathrm{g\ cm^{-1}\ sec^{-1}}, \tag{14}$$

where A_i is the mass number, Z the ionisation number and $\ln \Lambda$ the Coulomb logarithm [39].

From this we arrive at

$$R_e = 5.18 \times 10^{-20} v\, d n_i A_i^{1/2} Z^4 \ln \Lambda / T^{5/2}\,, \tag{15}$$

where T is now measured in electron-volts, all other in cgs.

When a magnetic field is present, the value of μ depends both on the direction of the velocity and the direction along which the gradient of velocity is considered. To compute the viscosity parallel to $\boldsymbol{B}$, i.e. $\mu_\parallel$, resulting from gradients of $v_\parallel$ in a direction along $\boldsymbol{B}$ the preceding formulation may be used directly. When the velocity $\boldsymbol{v}$ is perpendicular to the magnetic field, the stress resulting from a gradient in a direction perpendicular both to $\boldsymbol{B}$ and $\boldsymbol{v}$ may be expressed in terms of the coefficient $\mu_\perp$. For a gas in which the magnetic field is very strong, $\mu_\perp$ is given by Lehnert [19]:

$$\mu_\perp = 3\, n_i\, k\, T_i\, \nu_{ii}/16\, \omega_i^2\ \mathrm{g\ cm^{-1}\ sec^{-1}} \tag{16}$$

or, by elimination of ν_{ii} and ω_i we get (Spitzer [39])

$$\mu_\perp = 2.68 \times 10^{-2} \cdot (A_i^{3/2} Z^2 n_i^2 \ln \Lambda / T_i^{1/2} B^2)\ \mathrm{g\ cm^{-1}\ sec^{-1}}. \tag{17}$$

The Reynold's number, (11), is then

$$R_e = 6.68 \times 10^3 v\, d T_i^{1/2} B^2 / A_i^{1/2} Z^2 n_i \ln \Lambda\,, \tag{18}$$

where T is again in eV, all others in cgs.

Table 1 gives the relevant parameters for the experiments performed by Boeschoten [13], Döbele

	Döbele	Boeschoten	Wijnakker	
			Ions	Neutrals
T (eV)	4.31	3.7	0.16	0.16
A_i (g mol^{-1})	1.00	39.95	39.95	39.95
n_i (cm^{-3})	1.5×10^{15}	3.5×10^{12}	2.2×10^{13}	4.5×10^{15}
d (cm)	0.3	2.5	1	1
$\ln \Lambda$	6	11	6	—
v_i (cm S^{-1})	2.8×10^6	1.6×10^5	1×10^4	4×10^4
B (kG)	10	3.4	1.3	—

Table 1. Data for Reynold's number calculations. 1. values for $\ln \Lambda$ are from Spitzer [39]. 2. d chosen arbitrarily (e.g. 2.5 cm is the distance of the turbulence point to the wall for the Boeschoten case). 3. $Z = 1$, i.e. single ionisation.

Table 2. Reynold's numbers. In the neutrals calculation for Wijnakker a μ_n value from Fig. 7 was used.

	Döbele	Boeschoten	Wijnakker
no $\boldsymbol{B}$ or $\boldsymbol{v} \parallel \boldsymbol{B}$ (eq. (15))	10.2	0.19	42.2
$\boldsymbol{v}_\perp \boldsymbol{B}$ (eq. (18))	1.29×10^2	2.44×10^2	5.41×10^{-2}
neutrals (eq. (19))	—	—	14.6
ion-neutrals (eq. (18))	—	—	$\sim 5 \times 10^{-4}$

and others [17], and Wijnakker [9]. Table 2 gives the Reynold's numbers (remembering that for the case of no magnetic field and for that of $\boldsymbol{v} \parallel \boldsymbol{B}$, the Reynold's number will be identical).

For the case of Döbele, the hollow arc used has a radial dimension much smaller than that of the confining wall of the vessel. Figure 4 shows the radial velocity profile. It can be seen that for distances greater than 1.25 cm we are outside the arc and the gas is effectively at rest. For our calculations we take the thickness d to be the distance from the maximum at $r = 1.0$ cm to the radial limit of the arc (which effectively acts like a wall) so that $d \sim 3$ mm. Other parameters are shown in Table 1.

Which form for μ do we use in the case of a rotating plasma? In all the above, the magnetic field is axial with the rotation being about the axis. Hence we should expect to use $\mu_\perp$, that is (17).

Looking at the numbers in Table 2 this would appear to be correct. Here, in the case of $\boldsymbol{v}_\perp \boldsymbol{B}$, the Reynold's number for Boeschoten and Döbele is significantly greater than that for the other two.

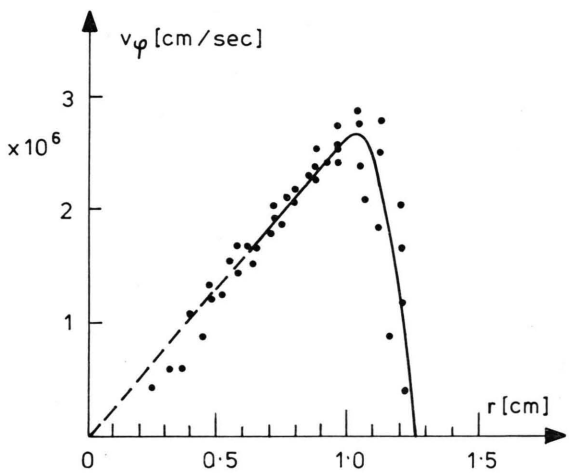

Fig. 4. Radial velocity profile (after Döbele [42]).

This result is desirable, as the density profile for Wijnakker exhibited no kinks, thus we probably still have laminar flow (which implies a relatively low Reynold's number).

On the other hand, using the field free formulation (which is equivalent to using $\mu_\parallel$), all cases gave low Reynold's numbers which would imply smooth, non-turbulent flow for all experiments.

Using this reasoning, it is obvious that the equation using $\mu_\perp$ gives more explainable results. However, this is not completely correct. It should be recognised that the Wijnakker experiment used only a partially ionised plasma being in the order of 1–2% for the diffuse mode. As such, the use of (17) is highly questionable.

Instead we now consider the contribution to the viscosity from the neutrals alone. Lehnert [11] quotes this to be

$$\mu_n = n_n k U_n / 3 \nu_{nn}, \tag{19}$$

where ν_{nn} is the neutral-neutral collision frequency. A rigorous treatment given by Hirschfelder et al. [41] gives the viscosity in terms of collision integrals $\Omega^{(1,s)*}$:

$$\mu_n = \frac{5}{16} \left(\frac{m_n k T_n}{\pi} \right) \frac{f_n}{\sigma^2 \Omega^{(2,2)*}}, \tag{20}$$

where σ is the collision cross section and f_n is a correction factor differing from unity by about 1%.

Dymond [42] evaluated the viscosity for the rare gases, neon to xenon. His results for Argon are shown in Figure 5. Using this information, the viscosities $\mu_\perp$ (17) and μ_n (19) may be compared for the results of Wijnakker.

We find that $\mu_\perp$ gives a marginally lower value of 2.7×10^{-4} gm cm^{-4} s^{-1} compared to that of 8.3×10^{-1} gm cm^{-1} s^{-1} for μ_n. The contributions to the viscosity are approximately equal. Thus the Reynold's number for the neutrals given in Table 2, of 14.6, is again low, implying we are in a laminar flow regime.

The effect of the ion-neutral collisions is more difficult to assess. Vrba [23] gives a collision frequency of approximately $1.43 > 10^{-9}$ n$_0$ s^{-1} at a temperature of $2 \times 10^3 \varphi$K. If this is compared to (13) we see that ν_{in} is by about a factor of 10^2 greater than ν_{ii}. Now, if we make the assumption of a form for the viscosity being that of (16) with ν_{ii} replaced by ν_{in}, this leads to a corresponding drop of the Reynold's number by a factor of 10^{-2}.

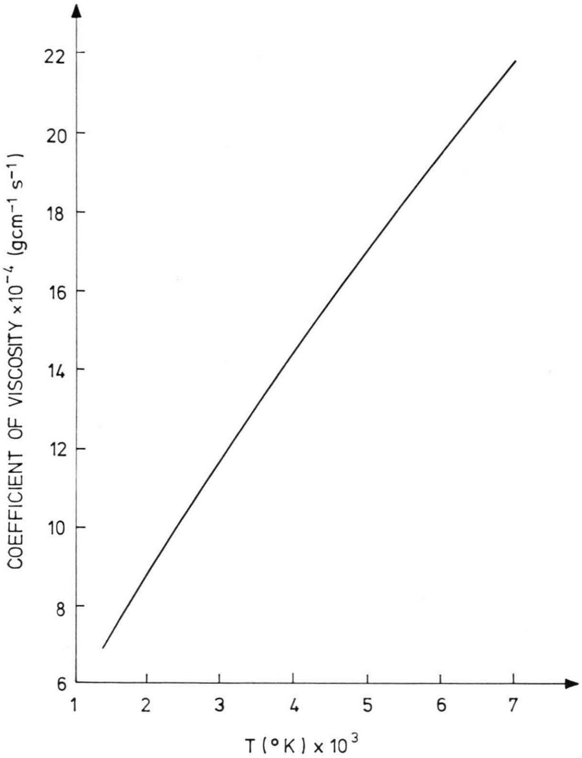

Fig. 5. Coefficient of viscosity vs. temperature for neutral argon (after Dymond [42]).

A complete description would have to include such parameters as the electron-ion, the neutral-ion and the neutral-neutral collision frequencies ν_{ei}, ν_{ni} and ν_{nn}, respectively. Also the role of the electric field that is used to put the plasma into rotation would need to be investigated. The $\boldsymbol{E} \times \boldsymbol{H}$ force would probably give some contribution to the viscosity.

From this simple discussion of turbulence from a hydrodynamic viewpoint, it would appear that the pessimism expressed by Boeschoten [13] — with regard to his own experiment — is largely unjustified. The low isotope separation factors achieved by him may have been due to the hydrodynamic turbulence discussed, and it is obvious that an adjustment of the relevant parameters is all that is needed to reduce R_e and hence the likelihood of turbulence. Hopefully this would improve the isotope separation, as the smoother laminar flow would negate the formation of small scale eddies, which decrease microscopic, diffuse separation effects.

5. Experimental Results

One of the first efforts to separate isotopes by rotating plasmas was made by Bonnevier [12]. This used a short lived plasma of about 1 ms duration which was formed by discharging a capacitor bank through a gas mixture. Using H_2/D_2 or H_2/Ar mixtures, experimental verification was found for a mass separation effect. His experiments are difficult to interpret as they were made in a complicated toroidal geometry — the plasma rotating like a car tyre. A similar but from the point of patent rights, sufficiently different configuration was developed by Hora and Karger [5] (Figure 6). In these cases, the pulsed operation does not allow an interpretation in stationary state terms. Cairns [43] performed a similar experiment using neon. A peak flow velocity of 1.3×10^5 cm s^{-1} was found for a B_z field of 2×10^3 G. An $\sim 15\%$ enrichment of Ne^{22} was achieved at a power input of $\sim 3.8 \times 10^{-8}$ kgms/kWhr.

James and Simpson [44] made experiments with a rotating plasma in the Supper III and later the Supper V plasma source. A rotating plasma was produced by discharging two capacitor banks between the concentric electrodes. The resulting radial

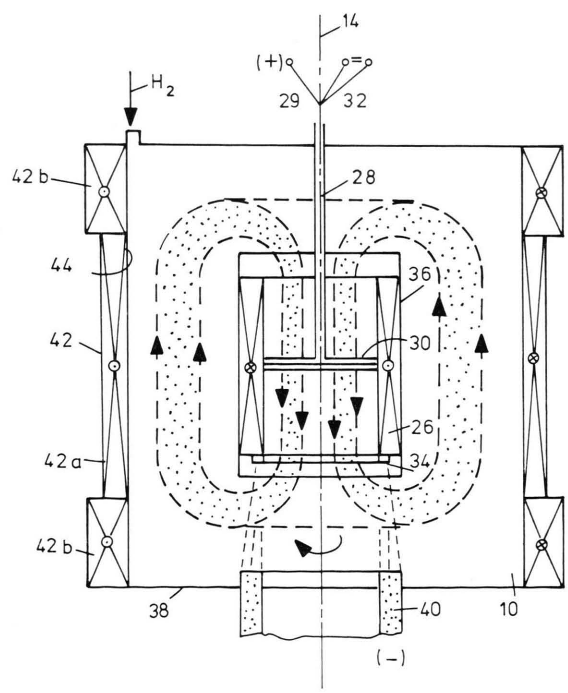

Fig. 6. U^{235} isotope separator of Hora and Karger [5].

current both ionized the gas and interacted with the axial field to produce an azimuthal force which rotated the plasma. The two banks gave a 12 kA, 50 μs current pulse, intended primarily to ionize the plasma, followed by a 3 kA pulse which increased both the ionization and plasma rotational velocity. Depending on the strength of the magnetic field, enrichment or depletion of Ne^{20} was observed. Apparently a complicated chain of events occurs in this machine and the properties of the plasma are not known sufficiently to determine optimum conditions for isotope separation. Nevertheless, it was possible to obtain separation factors around 1.2.

Later experiments by Brand, James and Walsh [45] examined the separation of metals. Two metallic elements, copper and nickel, are introduced into an argon plasma that is already rotating, and their separation is observed.

The cylindrical arc, pin and ring electrode device, suggested by Nathrath [46], was investigated for neon isotope separation by Heller and Simon [15]. Using this device, which operated continuously, separation factors of up to 1.11 were obtained.

The experiment of Wijnakker [9] (already partly discussed in the turbulence section of this paper) achieved a separation factor of 2.15 for argon and xenon while operating in a diffuse mode. Fig. 7 shows a similar configuration to that of Nathrath, except that we now have two hollow cathodes and two anodes. The length of the device between the two cathodes is 48 cm, the diameter in the region between the anodes 12 cm. The maximum current which can be drawn is 100 A per source. A homogeneous axial magnetic field with a strength up to

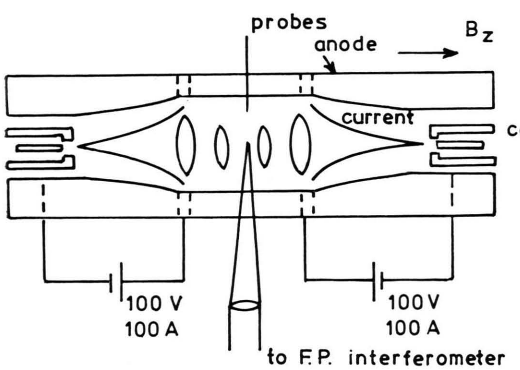

Fig. 7. Schematic view of Wijnakker's experiment. In the symmetry plane between the two anodes, diagnostic ports are present (after Wijnakker et al. [9]).

2.6×10^3 G can be applied externally, and during operation the pressure is kept constant at 1 torr in the volumes behind the hollow cathodes.

The hollow cathode discharge is an arc which burns in a vacuum environment made possible by the presence of a magnetic field along the axis of the arc. This was first realized by Luce [47]. Gas — in Boeschoten's case [13] argon — is introduced through an incandescent hollow tube at the cathode side and pumped away by vacuum pumps which maintain a vacuum in the range 10^{-4} to 10^{-3} torr. The gas is very efficiently ionized by passing the glowing hollow cathode.

The operating parameters for Boeschoten's device have been already discussed in the previous section. It was found that the ion temperature and the rotational velocity decrease along the axis, so that axial effects could not be neglected. The high rotational speeds observed in the plasma column did not lead to high separative powers. This is caused by the high temperature of the plasma and its associated counteracting effect on the radial diffusion of ions of different mass.

Kaneko et al. [10] performed an experiment on the centrifugal separation of an He-Ar gas mixture in a weakly ionized rotating plasma. The ratio Ar to He at the outer electrode increased with increasing axial magnetic field or radial current and reached a level 1.55 times larger than that before rotation when the magnetic field is 7.5 kG and the total electric current is 1.0 A. This result did not depend on the gas pressure from 0.6 to 20 torr.

6. The MHD-Driven Isotope Ultracentrifuge

Without a doubt, the renewal of interest in an electromagnetically driven isotope separator is due to its expected use as a separator of uranium isotopes for use in nuclear power stations. Indeed, Swedish research by Lehnert, Bonnevier and others at the Royal Institute of Technology in Stockholm is largely an offshoot of their nuclear fusion studies. Enrichment of isotopes by rotating plasmas has been achieved in several experiments, e.g. by Boeschoten and Nathrath [3], Bonnevier [12] or Ustinov et al. [48]. The last work is an original succes to realize an axial convection for increasing the separation effect as it was described by Hora and Karger [5]. While in the last mentioned case the convection is driven by the geometry and the hydrodynamic constellation of the configuration, Ustinov et al. [48] actively drive this convection by a counterexcitation of the plasma. Further modifications of this concept were discussed by Babaritskii et al. [48] using an inhomogeneous heating of the mixture of the components while special attention was given to the mechanisms of dissociation and excitation by electron impact by Ivanov [48].

The standard means of enrichment of the fissionable U^{235} isotope is by many stage diffusion of low pressure, gaseous UF_6 which requires a great amount of capital as well as a high power outlay. Upwards of a third of the eventual power production of the enriched uranium is consumed in the diffusion separation process.

A second type of isotope separation is the mechanical centrifuge process, which is under active development at Lucas Heights, in Europe, Japan and the U.S.A. The development of light weight casing centrifuges has enabled rotational speeds in excess of 5×10^4 r.p.m. to be achieved [49]. UF_6 gas at near one atmosphere is pumped in axial counterflow streams through a volume surrounded by the rotating cylinder.

In order to compare the different modes of isotope separation, it is sufficient to evaluate individual values of α, the separation factor, which is defined by [50], [51]

$$\alpha = \frac{N'/(1 - N')}{N''/(1 - N'')}, \tag{21}$$

where N' is the mole fraction of the desired isotope at the enriched (heads) side of a cascade stage, and N'' is the desired isotope mole fraction at the depleted (tails) side.

The gaseous diffusion α value is

$$\alpha^{\mathrm{D}} = (m_2/m_1)^{1/2} = \left(1 + \frac{\Delta m}{m_1}\right)^{1/2}, \tag{22}$$

where Δm is the isotopic mass difference.

The mechanical centrifuge α value is

$$\alpha^{\mathrm{M}} = \exp\left[(\Delta m)\, v_w^2/2\, R\, T\right] \tag{23}$$

where v_{w} is the wall tangential velocity, and R is the gas constant.

The relative fraction of the density distributions of two isotopes is determined to a large extent only by the centrifugal force. This means that (to a good approximation) the separation factor α^{MHD} for the plasma centrifuge is described by the same formula as that for the mechanical centrifuge [8].

α^{D} is fixed at 1.0043 for UF_6. At a peripheral speed of 5×10^4 cm s^{-1} and a temperature of 310 K we get α^{M} equal to 1.156 [52], The obvious benefit of the plasma centrifuge is the order of magnitude greater peripheral speeds.

The separative power of a centrifuge also depends very strongly on the rotational velocity [52]:

$$\delta U_{\max} = \frac{\pi}{2}\, Z_H\, \varrho\, D\left(\frac{(\Delta m)\, v_w^2}{2\, R\, T}\right)^2, \tag{24}$$

where δU is the separative power and Z_{H} the height of the centrifuge. From this we see that the separative power increases with the fourth power of the rotational velocity.

Kane and George [37] showed that the required number of stages in an enrichment cascade, to achieve a desired enrichment level, is significantly less for a plasma centrifuge than that for membrane diffusion or mechanical centrifuge.

Uranium 235 is contained in natural uranium at a level of 0.7%. It is necessary to enrich this content to 3% for nuclear power generation. This enrichment process is usually expressed in terms of "separative work units" (SWU). SWU is a measure [53] of the separative work necessary to produce P kg of product at composition N_p, when the plant is fed with F kg of mixture at composition N_F and rejects W kg of mixture at composition N_W.

As so little experimental data on rotating uranium plasmas is available, recent analysis has been restricted to numerical studies [10], [21], [24].

Using plasma parameters described earlier, Okada and Dodo calculated an equilibrium separation factor of 1.25. Thus only 13 stages are sufficient for raising the concentration from 0.7% to 3% at a separative work of 7.4×10^{-4} g SWU/sec. Raising the concentration to 3% by a single stage can be realised if the equilibrium separation factor can be made to reach 4.3. The power consumed per unit product quantity is 1.16 kW/kg SWU/year which is about ten times that of the mechanical centrifuge. The above figure may not be that bad though, as the system studied was not optimised.

Kaneko et al. [10] estimated that 10 stages are needed to raise the concentration from 0.7% to 3% at an energy efficiency of 14 MeV/U^{235}.

Wilhelm and Hong [21] estimate a separation factor of 1.13 at a rotational velocity of 1×10^5 cm $\cdot$ s^{-4}. They also proposed a scheme for a multidischarge centrifuge. Using a discharge such as that of Fig. 7, a large number of such discharges can be set up in a long insulating cylinder to rotate large volumes of isotope mixtures. Each circumferential cathode or anode serves as a common electrode for the adjacent discharges. It is seen that the volume of rotating plasma is nearly doubled in each electrode region, compared to the single discharge centrifuge. This would be located at some stage in a cascade, the enriched and depleted isotope streams are introduced at one end of each centrifuge stage and removed at the other end.

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